1 Multivariate discrete random variables

Let $X : \Omega \to \mathbb{Z}$ and $Y : \Omega \to \mathbb{Z}$ be discrete random variables with joint PMF

$$p(x, y) = \begin{cases} c\,xy & \text{if } 1 \leq x \leq y \leq 3 \\ 0 & \text{else} \end{cases}$$

1. Find the normalizing constant c.
2. Are X, Y independent? Prove your claim.
3. Find the expectations of X,Y, XY.

2 Multivariate continuous random variable

Let (X, Y) be two continuous RVs with joint density

$$\rho(x, y) = \begin{cases} cxy & \text{if } x, y \in [0, 1] \\ 0 & \text{else} \end{cases} .$$

1. Find the normalizing constant c.
2. Find the marginal densities and expectations of X,Y.
3. Find the conditional expectation of X given $Y = y$.