Machine Learning

Spring Semester 2021
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Assignment Sheet 1.   Submit on **Tuesday, February 16, 2020, 10:00**.

Excercise 1. (Random experiments)

The disjoint events A, B, and C are defined in a sample space Ω. Find expressions for the following probabilities in terms of sums the probabilities $P(\emptyset), P(A), P(B), P(A \cup B), P(A \cup C), P(B \cup C), P(A \cup B \cup C)$:
(Example: the probability that A or B occurs: $P(A) + P(B)$)

a) the probability that exactly two of A; B; C occur *(at the same time!)*;

b) the probability that exactly one of these events occurs;

(4 Points)

Excercise 2. We consider the chance experiment of four tosses of a fair coin.

a) Give the sample space Ω for this experiment and define a discrete random variable X to describe the number of heads obtained in four tosses of a fair coin.

b) Find the PMF and CDF of the defined random variable and plot the CDF.

c) Compute the probability of the event X IS BIGGER THAN 1

d) Compute the mean of the random variable.

(4 Points)

Excercise 3. Let the function ρ with $\rho(x) = \begin{cases} c|x|(1+x)(1-x) & \text{if } -1 \le x \le 1, \\ 0 & \text{else,} \end{cases}$ be given.

a) Compute the constant c, such that ρ becomes a density of some random variable.

b) Consider a random variable X with the just computed density ρ. Find its CDF and plot it.

c) Use the CDF to compute

$$P(X < -0.5), \quad P(X > 0.5), \quad P(-0.5 < X < 0.5).$$

(4 Points)

Programming Exercise 1. In this first programming exercise, we would like to familiarize us with typical linear algebra computing tasks. To this end, we define the vectors / matrices

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}, \quad M = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Students are free to choose between the programming languages Python and C/C++. In case of Python, the module *NumPy* should be used for the linear algebra tasks. For C++, there exists a convenient numerics header-only library called *Eigen*. Classical C programmers can go for *LAPACK*. (Reference solutions will only be provided in Python.) The submission format for Python is a Jupyter notebook. The submission format for C/C++ is standard source files.

Use one of the listed programming languages with the respective library to implement the following tasks:

a) Compute and print the inner product between $\mathbf{a}$ and $\mathbf{b}$.

b) Compute and print the matrix-vector product between M and $\mathbf{b}$.

c) Compute and print the l_2 norm of $\mathbf{b}$.

d) Compute and print the the solution $\mathbf{x}$ of the linear system of equations $M\mathbf{x} = \mathbf{b}$ using the LU factorization.

e) Compute and print the the solution x of the linear system of equations $M\mathbf{x} = \mathbf{b}$ using the Cholesky factorization.

(4 Points)